Solution to MHT CET – 2021
23rd September (Shift - 1)

Section I

PHYSICS

1. (A)

$C = \sqrt{\frac{3RT}{M}}$

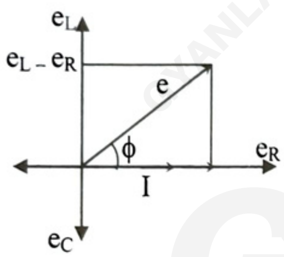
$\therefore \quad C \propto m^{-\frac{1}{2}} T^{\frac{1}{2}}$

2. (A)

$g = \frac{GM}{R^2}, \; g' = \frac{GM'}{R^2} = \frac{G \times 4M}{R^2} = 4\,g$

$W = mg'h = 4mgh = 4 \times 5 \times 10 \times 2 \; = 400 \text{ J}$

3. (A)



(A) is correct

(B) is wrong. There is phase difference ϕ between the applied emf e and e_R

(C) is wrong. There is phase difference of $\left(\frac{\pi}{2} - \phi\right)$ between e and e_L

(D) is wrong. There is a phase difference of π^c between e_C and e_L

$\therefore$ B, C and D are wrong.

4. (A)

$y_1 = 0.25 \sin 316t, \; y_2 = 0.25 \sin 310t$

$\therefore \quad 2\pi f_1 = 316 \;$ and $2\pi f_2 = 310$

$\therefore \quad f_1 = \frac{316}{2\pi} = \frac{158}{\pi}$

$f_2 = \frac{310}{2\pi} = \frac{155}{\pi}$

$\therefore$ Beat frequency $= f_1 - f_2$

$= \frac{158}{\pi} - \frac{155}{\pi} = \frac{3}{\pi}$

5. (C)

The voltage across L and C has a phase difference of π^c.

Hence the resultant voltage across the combination is zero.

6. (C)
Terminal velocity $V \propto r^2$

$\therefore \frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\therefore V_2 = \frac{V_1}{4} = \frac{20}{4} = 5$ cm/s

7. (D)
$F = qE = 1.6 \times 10^{-19} \times 5 \times 10^{11}$
$= 8 \times 10^{-8}$ N

8. (C)
$E = \frac{1}{2}kx^2; \quad F = -kx$

$\therefore 2E = kx^2$

$\therefore \frac{2E}{F} = -x$

$\therefore \frac{2E}{F} + x = 0$

9. (A)
$E = \frac{1}{2}I\omega^2$

$I = 2m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$

$\therefore E = \frac{1}{2} \times \frac{md^2}{2} \cdot \omega^2 = \frac{md^2}{4} . \omega^2$

$\therefore \omega^2 = \frac{4E}{md^2}$

$\therefore \omega = \frac{2}{d}\sqrt{\frac{E}{m}}$

10. (C)
Magnifyting power $m = \frac{f_o}{f_e}$

If f_e is doubled, the magnifying power will become $\frac{m}{2}$

11. (B)
$m_1 = m, \ u_1 = v, \ \vartheta_1 = 0$
$m_2 = 4m, \ u_2 = 0, \ \vartheta_1 = ?$
By law of conservation of momentum we have
$m_1u_1 + m_2u_2 = m_1\vartheta_1 + m_2\vartheta_2$
$\therefore mv + 0 = 0 + 4m_2\vartheta_2$
$\therefore v = 4\vartheta_2 \quad$ or $\quad \vartheta_2 = \frac{v}{4}$

Coefficient of restitution $e = \frac{\vartheta_2 - \vartheta_1}{u_1 - u_2}$

$$= \frac{\frac{v}{4} - 0}{v - 0} = \frac{1}{4} = 0.25$$

12. (B)

Since the spheres are joined by a metal wire, their potentials will be same.
Let the radius of the smaller sphere be r_1
and the radius of the larger sphere be $r_2 = 4r_1$

the $v = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2}$

$$\therefore \frac{q_2}{q_1} = \frac{r_2}{r_1}$$

The electric field near the surface of spheres are given by

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1^2} \text{ and } E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2^2}$$

$$\therefore \frac{E_2}{E_1} = \frac{q_2}{q_1} \cdot \frac{r_1^2}{r_2^2} = \frac{r_2}{r_1} \cdot \frac{r_1^2}{r_2^2} = \frac{r_1}{r_2} = \frac{1}{4}$$

$$\therefore E_2 = \frac{E_1}{4}$$

13. (B)

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$\therefore h \propto \frac{1}{r}$$

$$\therefore \frac{h_2}{h_1} = \frac{r_1}{r_2}$$

Area $A = \pi r^2$ $\quad \therefore A \propto r^2$

$$\therefore \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \text{ or } \frac{r_1}{r_2} = \sqrt{\frac{A_1}{A_2}} = \sqrt{16}$$

$$\frac{r_1}{r_2} = 4$$

$$\therefore \frac{h_2}{h_1} = 4 \quad \text{or} \quad h_2 = 4h_1 = 4 \times 2 = 8 \text{ cm}$$

14. (C)

Escape velocity $v = \sqrt{\frac{2GM}{R}}$

$$M = \frac{4}{3}\pi R^3 \cdot \rho$$

$\therefore \quad v=\sqrt{\frac{2G\times\frac{4}{3}R^3\cdot\rho}{R}}=\sqrt{\frac{8G}{3}R^2\rho}=R\sqrt{\frac{8G}{3}\rho}$

$\therefore \quad v\propto R\sqrt{\rho}$

$\therefore$ If ρ is constant, the $v \propto R$

$\therefore \quad \frac{v_p}{v_E}=\frac{R_p}{R_E}=2$

$\therefore \quad v_p=2v_E$

15. (C)
Shortest wavelength in Lyman series is given by

$$\frac{1}{\lambda_L}=R\left[\frac{1}{1^2}-\frac{1}{\infty}\right]=R$$

$$\therefore \quad \lambda_L=\frac{1}{R}$$

The longest wavelength in Paschen series is given by

$$\frac{1}{\lambda_p}=R\left[\frac{1}{(3)^2}-\frac{1}{(4)^2}\right]$$

$$=R\left[\frac{1}{9}-\frac{1}{16}\right]=R\cdot\frac{7}{144}$$

$$\lambda_p=\frac{144}{7R}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_L}=\frac{144}{7R}\cdot R=\frac{144}{7}$$

$$\therefore \quad \lambda_p=\frac{144}{7R}\cdot\lambda_L=\frac{144}{7}\times 912=18760 \text{ Å}$$

16. (B)
Wire has a resitance $1\ \Omega\ cm^{-1}$.
$\therefore$ Resistance of 40 cm of wire = 40 Ω
and reistance of 60 cm of wire = 60 Ω
No current flows through the galvanometer since the bridge in balanced. Hence the branch containing the galvanometer can be removed.

For balanced bridge $\frac{4}{40}=\frac{Y}{60}$

$\therefore \quad Y=\frac{4\times 60}{40}=6\,\Omega$

4 Ω and 6 Ω are in series. Their equivalent resistance is 10 Ω.
40 Ω and 60 Ω are in series. Their equivalent resistance is 100 Ω.
10 Ω and 100 Ω are in parallel
Their equivalent resistance is $\frac{1000}{110}\,\Omega$.

$\therefore$ The current $I=\frac{V}{R}=\frac{6\times 110}{1000}=0.66\text{ A}$

17. (D)

With only one slit there will be no interference and no fringes will be observed.

18. (B)

$X = \frac{\vartheta_a}{\vartheta_w}, \quad Y = \frac{\vartheta_w}{\vartheta_g}; \quad Z = \frac{\vartheta_g}{\vartheta_a}$

$\therefore \quad XYZ = 1$

19. (D)

20. (C)

$\frac{E_1}{E_2} = \frac{\ell_1 + \ell_2}{\ell_1 + \ell_2} = \frac{64+32}{64-32} = \frac{96}{32} = 3$

21. (A)

Moment of inertia of the upper sphere = $\frac{2}{5}MR^2$

For each lower sphere M.I. $= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

$\therefore$ Total moment of inertia $I = \frac{2}{5}MR^2 + 2\times\frac{7}{5}MR^2 = \frac{16}{5}MR^2$

22. (B)

Kinetic energy is maximum at the mean position and potential energy is maximum at the extreme positions on either side,

The distance between mean and extreme positions is A.

23. (D)

24. (B)

Truth table :

A	B	C	D	Y
0	0	1	1	0
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

This is truth table of AND gate.

$\therefore \quad Y = A \cdot B$

25. (B)

Let K represent kinetic energy

$\therefore \quad K_1 = \frac{hc}{\lambda_1} - W_0$

and $\quad K_2 = \frac{hc}{\lambda_2} - W_0$

$K_2 = 3K_1$

$\therefore \quad \frac{hc}{\lambda_2} - W_0 = 3\frac{hc}{\lambda_1} - 3W_0$

$\therefore \quad 2W_0 = \frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2}$

$\therefore \quad W_0 = \frac{hc}{2}\left(\frac{3\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)$

26. (A)
At the highest point v = 0

$\therefore \ 0 = u - gt$

$\therefore \ u = gt = 10 \times 3 = 30$ m/s

Also $0 = u^2 - 2gh$

$\therefore \quad h = \frac{u^2}{2g} = \frac{(30)^2}{2\times10} = 45$ m

27. (A)

$W = 8\pi T\left(r_2^2 - r_1^2\right)$

$= 8\pi\times0.03(25-9)\times10^{-4}$

$= 0.384\pi \times 10^{-3}$ J

$= 0.384\,\pi$ mJ $\approx 0.4\pi$ mJ

28. (C)
In circuit X, both the diodes are forward biased and hence both will conduct.
The two resistances of 4 Ω each are in parallel. Their equivalent resistance is 2 Ω. Hence the current $I = \frac{V}{R} = \frac{8}{2} = 4A$

In the circuit Y, the diode D_1 is forward biased but diode D_2 is reverse biased. Hence only diode D_1 will conduct. The resistance is 4 A.

Hence $I = \frac{8}{4} = 2$ A

29. (B)

$W = \frac{\lambda D}{d}$

$\therefore \ \frac{W_2}{W_1} = \frac{D_2}{D_1}\cdot\frac{d_1}{d_2}$

$D_2 = 1.25D_1$ and $d_2 = \frac{d_1}{2}$

$\therefore \ \frac{W_2}{W_1} = 1.25\times2 = 2.5$

$\therefore \ W_2 = 2.5$ W

30. (B)

$y_1 = A\sin(\omega t + kx + 0.57)$

$y_2 = A\cos(\omega t + kx)$

$= A\sin(\omega t + kx + \frac{\pi}{2})$

$\therefore$ Phase difference $= \frac{\pi}{2} - 0.57$

$= \frac{3.14}{2} - 0.57$

$= 1.57 - 0.57 = 1.0$ rad

31. (D)

Since density becomes twice, its volume will become half.

$\therefore \frac{V_1}{V_2} = 2$

$\therefore$ For adiabatic process we have

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (2)^{7/5} = (2)^{1.4}$$

$= 2.63$

32. (D)

$$\beta = \frac{\Delta I_C}{\Delta I_b} = \frac{5}{0.2} = 25$$

Voltage gain $A_v = \beta \frac{R_L}{R_i}$

$$\therefore R_L = \frac{A_v R_i}{\beta} = \frac{75 \times 2}{25} = 6\,k\Omega$$

33. (C)

The moment of inertia is given by

$$I = \sum m_i r_i^2$$

If the distance of particles from the axis of rotation is larger, then the moment of inertia will be larger. For axis QR, the particles will be situated at greater distances and hence the moment of inertia will be greater.

34. (C)

35. (C)

The energy in ground state (n = 1) is 13.6 eV

The energy in n = 2 state is $\frac{-13.6}{4} = -3.4$ eV

Angular momentum $= \frac{nh}{2\pi} = \frac{2h}{2\pi} = \frac{h}{\pi}$

36. (C)

Coefficient of emission = coefficient of absorption

For a perfectly black body a = e = 1

37. (D)

For same change in temperature the change in length is same for both the rods.

Change in length $\Delta L = L\alpha\,\Delta T$

$\therefore\; L_1\,\alpha_1\,\Delta T = L_2\,\alpha_2\,\Delta T$

or $L_1\,\alpha_1 = L_2\,\alpha_2$

38. (C)

Rate of radiation $R \propto T^4$

$$\therefore\; \frac{R_2}{R_1} = \left(\frac{T_2}{T_1}\right)^4 = (1.5)^4 \approx 5$$

$$\therefore\; R_2 = 5R_1$$

$$R_2 - R_1 = 4R_1$$

$$\frac{R_2 - R_1}{R_1} = 4$$

$$\text{Percentage increase} = \left(\frac{R_2 - R_1}{R_1}\right)\times 100 = 400\%$$

39. (D)

$$v = \omega\sqrt{A^2 - x^2}$$

$$v_{max} = A\omega$$

$$\therefore\; \omega\sqrt{A^2 - x^2} = \frac{A\omega}{4}$$

$$\therefore\; \sqrt{A^2 - x^2} = \frac{A}{4}$$

$$\therefore\; A^2 - x^2 = \frac{A^2}{16}$$

$$\therefore\; 16A^2 - 16x^2 = A^2$$

$$\therefore\; 16x^2 = 15A^2$$

$$\therefore\; x^2 = \frac{15}{16}A^2$$

$$\therefore\; x = \frac{A\sqrt{15}}{4}$$

40. (C)

Fundamental frequency of pipe closed at one end

$$n = \frac{v}{4\ell}$$

Second overtone of pipe open at both the ends

$$n' = 3\left(\frac{v}{2\ell'}\right)$$

$$n' = n$$

$$\therefore\; \frac{3v}{2\ell'} = \frac{v}{4\ell}$$

$$\therefore\; \frac{\ell}{\ell'} = \frac{1}{6}$$

41. (D)

The direction of the magnetic field will be along the axis. The angle between the velocity and the magnetic field will be 0° or 180°. The force is given by

$F = qvB \sin\theta$

If $\theta = 0°$ or $180°$, $F = 0$

42. (C)

For circular coil of radius R carrying a current I, the magnetic field at the centre is given by

$$B = \frac{\mu_0 I}{2R}$$

Here $I = qf$

$$\therefore \quad B = \frac{\mu_0 qf}{2R}$$

43. (D)

$$V = \frac{q}{4\pi\varepsilon_0 r}\left[1-\frac{1}{3}\right] = \frac{2}{3}\cdot\frac{q}{4\pi\varepsilon_0 r}$$

$$E = \frac{q}{4\pi\varepsilon_0 r}\cdot\frac{1}{(3r)^2} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{9r^2}$$

$$\frac{E}{V} = \frac{1}{6r} \quad \therefore \; E = \frac{V}{6r}$$

44. (B)

Distance of 4th dark band from the centre is given by $3.5\frac{\lambda D}{d}$

$$\therefore \quad \frac{d}{2} = \frac{3.5\lambda D}{d}$$

$$\therefore \quad \lambda = \frac{d^2}{7D}$$

45. (B)

$\phi = 3t^2 + 4t + 7$

$|e| = \frac{d\phi}{dt} = 6t + 4$

at $t = 2$ s, $|e| = 16$ V

46. (D)

$I = \frac{e}{R}$ or $e = IR$

$$e = \frac{d\phi}{dt} \quad \therefore \; \frac{d\phi}{dt} = IR = 1.5\times10^{-3}\times 5$$

$$= 7.5\times10^{-3} \text{ Wb/s}$$

47. (B)

Inductive reactance $X_L = 2\pi fL$

$\therefore \quad X_L \propto f$

48. (C)

If E is the kinetic energy of the electron then

$$E = \frac{p^2}{2m} \qquad (p = \text{momentum})$$

$\therefore \; p = \sqrt{2mE}$

$\therefore$ de Broglie wavelength of electron

$$\lambda_e = \frac{h}{\sqrt{2mE}}$$

Energy of photon $E = \dfrac{hc}{\lambda} \quad \therefore \lambda_p = \dfrac{hc}{E}$

$$\therefore \; \frac{\lambda_e}{\lambda_p} = \frac{h}{\sqrt{2mE}} \times \frac{E}{hc}$$

$$= \frac{1}{c}\sqrt{\frac{E}{2m}}$$

49. (C)

In the absence of the dielectric the electric field between the plates is given by

$$E = \frac{q}{A\varepsilon_0}$$

when dielectric is placed, the field gets reduced by the factor k.

The potential difference between the plates is given by

$$V = E_1t_1 + E_2t_2 + E_2t_3$$

$$= \frac{E}{k_1}\cdot t_1 + \frac{E}{k_2}\cdot t_2 + \frac{E}{k_3}\cdot t_3$$

$$= E\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}\right)$$

$$= \frac{q}{A\varepsilon_0}\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}\right)$$

$$C = \frac{q}{V} = \frac{A\varepsilon_0}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}\right)}$$

50. (D)

$L = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}^2$

$A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$

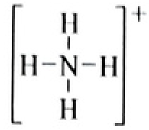
$M = 3 \text{ Am}^2$

$M_z \rightarrow$ Intensity of magnetisation

$$= \frac{M}{L \times A} = \frac{3 \text{ Am}^2}{3 \times 2 \times 10^{-6} \text{m}^3} = \frac{1}{2} \times 10^6 \text{ A/m} = 5 \times 10^5 \text{ A/m}$$

CHEMISTRY

51. (A)

NH_4^+ ion ⇒ 

$$FC = VE - NE - \left(\frac{BE}{2}\right)$$

∴ Formal charge of 'N' atom $= 5 - 0 - \left(\frac{8}{2}\right) \quad = +1$

52. (B)

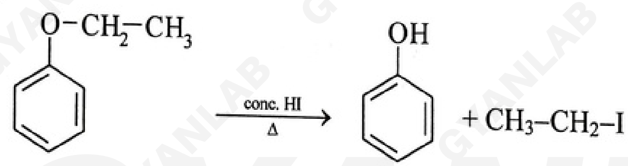
The observed stability order of alkyl free radicals is :

$(CH_3)_3C\cdot > (CH_3)_2CH\cdot > CH_3{-}CH_2\cdot > CH_3\cdot$

53. (A)

$CuCl_2$ is a salt of strong acid, HCl and weak base, $Cu(OH)_2$. Therefore, $CuCl_2$ salt solution is acidic and shows pH less than 7.

54. (A)



Ethoxybenzene Phenol Ethyl iodide

55. (D)

$W_2 = 1.8$ g, $M_2 = 180$ g mol^{-1}, $W_1 = 16.2$ g,

$M_1 = 18$ g mol^{-1}, $P_1^0 = 24$ mm Hg, $P_1 = ?$

$$\frac{P_1^0 - P_1}{P_1^0} = \frac{W_2 M_1}{M_2 W_1}$$

$$\therefore \frac{24\text{ mm Hg} - P_1}{24\text{ mm Hg}} = \frac{1.8\text{ g} \times 18\text{ g mol}^{-1}}{180\text{ g mol}^{-1} \times 16.2\text{ g}}$$

$\therefore$ 24 mm Hg $- P_1$ = 24 mm Hg (0.0111)

$\therefore$ P_1 = 24 mm Hg − 0.2664 mm Hg

$\therefore$ P_1 = 23.73 mm Hg

56. (B)

$r = k[A][B][C]^2$

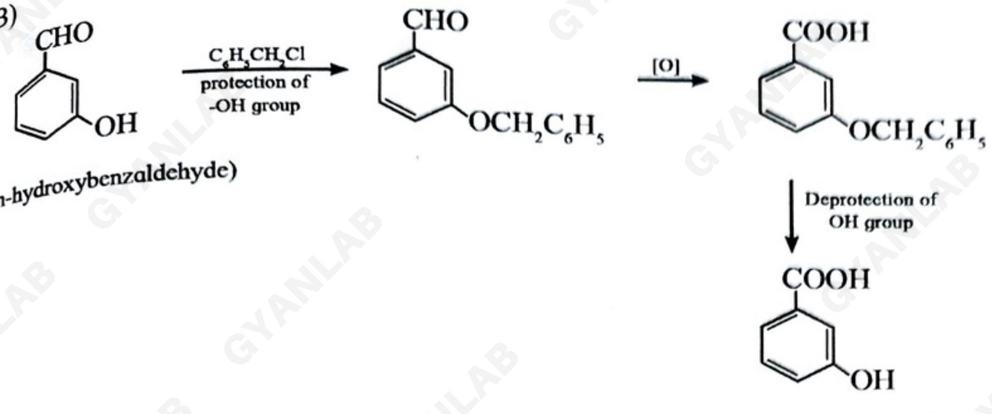
If concentration of both A and B are doubled, then new rate of reaction is

$r' = k\,[2A]\,[2B]\,[C]^2$

$= 4k\,[A]\,[B]\,[C]^2$

$\therefore r' = 4r$

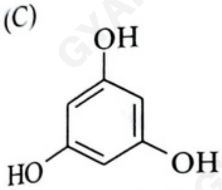
57. (B)



(m-hydroxybenzaldehyde)

(m-hydroxybenzoic acid)

58. (C)



Phorogucinol → It is trihydric phenol.

59. (A)

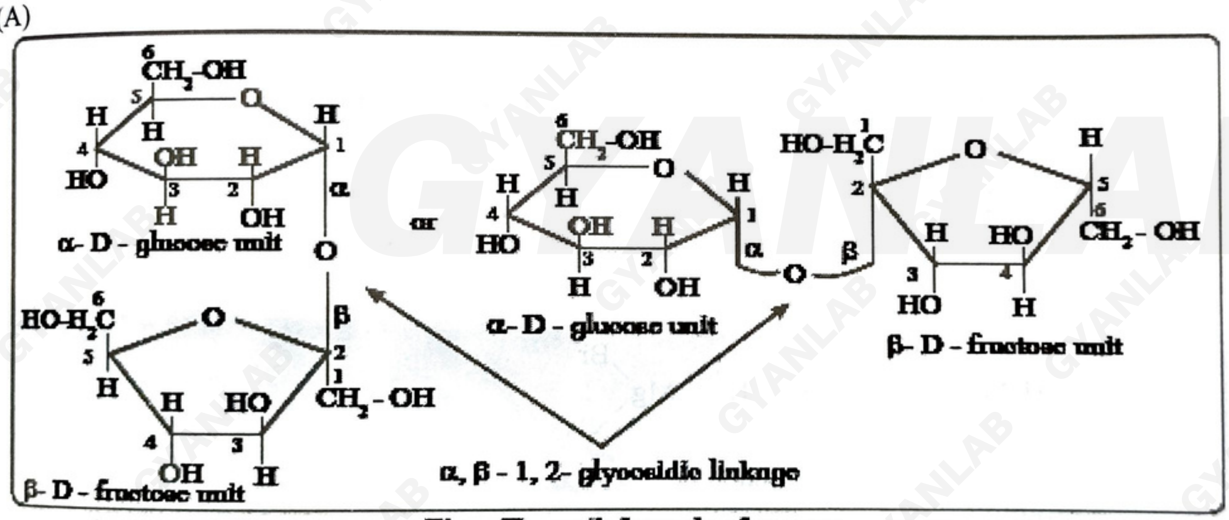


Fig. : Haworth formula of sucrose

60. (D)

$m = 1 \text{ mol kg}^{-1}$, $K_f = 1.86 \text{ K kg mol}^{-1}$, $T_f^o = 0\ °C$

$\Delta T_f = K_f m$

$= 1.86 \text{ K kg mol}^{-1} \times 1 \text{ mol kg}^{-1}$

$= 1.86 \text{ K} \quad = 1.86\ °C$

Now, $\Delta T_f = T_f^o - T_f$

$\therefore \quad T_f = T_f^o - \Delta T_f$

$= 0\ °C - 1.86\ °C \quad = -1.86\ °C$

61. (D)

62. (A)

Among transition metals Fe, Co, Ni are Ferromagnetic.

63. (A)

$$CH_3-CH=CH-CH_2-C\equiv N \xrightarrow[H_3O^+]{AlH(i-Bu)_2} CH_3-CH=CH-CH_2-CHO$$

Pent-3-enenitrile → Pent-3-enal

64. (D)

$[CoF_6]^{3-}$ → Paramagnetic and octahedral
$[Co(NH_3)_6]^{3+}$ → Diamagnetic and octahedral
$[NiCl_4]^{2-}$ → Paramagnetic and tetrahedral
$[Ni(CN)_4]^{2-}$ → Diamagnetic and square planar

65. (D)

$22.4\ dm^3$ of CH_4 = 16 g at STP

$$\therefore\ 33.6\ dm^3 \text{ of } CH_4 = \frac{16\times 33.6}{22.4}$$

$$= 24\ g \qquad = 2.4\times 10^{-2}\ kg$$

66. (A)

For BCC structure, n = 2 atoms

$a = 500\ pm = 5\times 10^{-8}$ cm, $\rho = 4\ g\ cm^{-3}$, M = ?

$$M = \frac{\rho a^3 N_A}{n}$$

$$= \frac{4\,g\,cm^{-3}\times(5\times 10^{-8})^3\,cm^3\times 6.022\times 10^{23}\text{ atoms mol}^{-1}}{2\text{ atoms}}$$

$$= \frac{4\times 125\times 10^{-24}\times 6.022\times 10^{23}}{2}$$

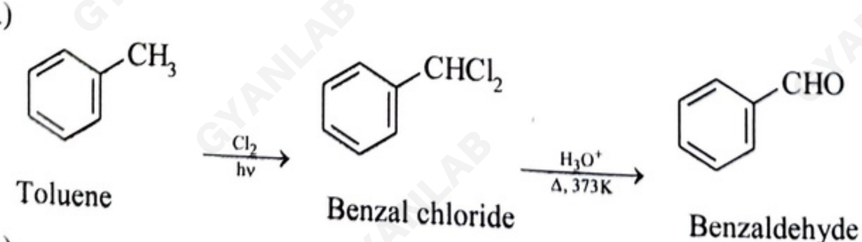
$$= 150.5\ g\ mol^{-1}$$

67. (A)

$$C_2H_5MgBr \xrightarrow{NH_3} C_2H_6 + Mg\begin{matrix}\diagup Br\\ \diagdown NH_2\end{matrix}$$

Ethyl magnesium bromide → Ethane

68. (A)



69. (A)

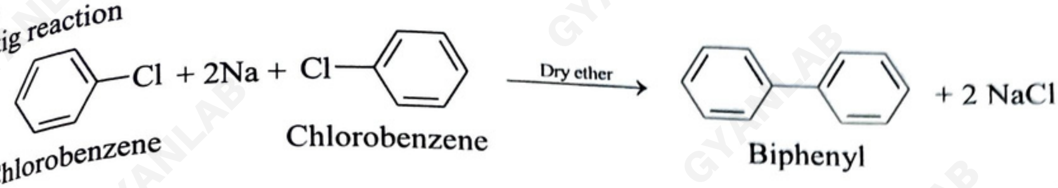
Q = –x J, W = –y J

According to first law of thermodynamics,

$$\Delta U = Q + W$$

$$\therefore\ \Delta U = -x - y\ J$$

70. (C)
Fittig reaction



71. (C)
$W = -394\ J,\ Q = 701\ J$
According to first law of thermodynamics,
$\Delta U = Q + W$
$= 701 - 394 \qquad = 307\ J$

72. (B)
$I = 1.5\ A,\ t = 3\ hr\ = 3 \times 60\ min \times 60\ s\ = 10800\ s$
$Q = It\ = 1.5 \times 10800\ = 16200\ C$
Now, $1.602 \times 10^{-19}\ C = 1\ e^-$

$\therefore \quad 16200\ C \ = \dfrac{1e^- \times 16200\,C}{1.6 \times 10^{-19}\ C}$

$= 10125 \times 10^{19}\ e^-$
$= 1.01 \times 10^{23}\ e^-$

73. (C)
$k = 0.0627\ S\ cm^{-1},\ c = 0.3\ mol\ L^{-1}$

$\wedge \ = \dfrac{1000\ k}{c}$

$= \dfrac{1000\,cm^3\,L^{-1} \times 0.0627\,S\,cm^{-1}}{0.3\ mol\ L^{-1}}$

$\therefore\ \wedge \ = 209\ S\ cm^2\ mol^{-1}$

74. (B)
$S = 2 \times 10^{-4}\ mol\ L^{-1},\ K_{sp} = ?$
$Ag_2C_2O_4 \rightleftharpoons 2Ag^+ + C_2O_4^{2-}$
$x = 2,\ y = 1$
$\therefore\ K_{sp} = 4s^3$
$= 4 \times (2 \times 10^{-4})^3 \quad = 3.2 \times 10^{-11}$

75. (D) **76.** (A)

77. (A)
HDP is obtained by polymerization of ethene in presence of Zieglar-Natta catalyst.

78. (D)

79. (A)
$V_1 = 3.4\ L,\ T_1 = 298\ K$
$V_2 = 6.8\ L,\ T_2 = ?$
According to Charle's law

$\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$

$$\therefore\ T_2 = \frac{V_2 \times T_1}{V_1}$$

$$= \frac{6.8\,L \times 298\,K}{3.4\,L} = 596\ K$$

80. (B)

a = 336 pm, r = ?

For simple cubic structure,

$$r = \frac{a}{2}$$

$$= \frac{336\ pm}{2} = 168\ pm$$

81. (B)

82. (B)

$$n\,CF_2 = CF_2 \xrightarrow[\text{Peroxide}]{\text{Polymerization}} -[-CF_2-CF_2-]_n-$$

Tetrafluoroethylene → Teflon

83. (C)

$$\underset{\text{Bromomethane}}{CH_3-Br} + 2Na + \underset{\text{Bromoethane}}{Br-CH_2-CH_3} \xrightarrow{\text{Dry ether}} \underset{\text{Propane}}{CH_3-CH_2-CH_3} + 2NaBr$$

Ethane and Butane are self-coupling products.

Methane can not be prepared by Wurtz reaction.

84. (B)

For first order reaction,

$$t = \frac{2.303}{k}\log_{10}\frac{[A]_0}{[A]_t}$$

Let t_1 be the time required for 90% completion and

t_2 be the time required for 99% completion.

$$\therefore\ \frac{t_1}{t_2} = \frac{\log_{10}\frac{100}{10}}{\log_{10}\frac{100}{1}} = \frac{1}{2}$$

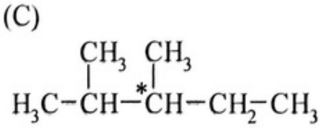
$$\therefore\ t_2 = 2t_1$$

85. (B)

$$\propto = 3.0\% = \frac{3}{100} = 3 \times 10^{-2},\ c = 0.04\ M$$

$$K_{\propto} = \propto^2 c = (3 \times 10^{-2})^2 \times 0.04 = 3.6 \times 10^{-5}$$

86. (C)



2,3-Dimethylpentane

87. (D)
Consider the diamagnetic octahedral complex, $[Co(NH_3)_6]^{3+}$. In this complex ion, oxidation state of cobalt is +3. It has electronic configuration as $3d^6$. This complex involves the d^2sp^3 hybridization. The hybridization scheme is shown in diagram.

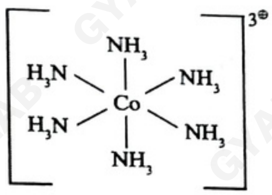
Orbitals of Co^{3+} ion: 3d [↑↓ | ↑ | ↑ | ↑ | ↑]   4s [ ]   4p [| |]

Since NH_3 is a strong ligand, due to spin pairing effect, all the four unpaired electrons in 3d orbital are paired giving two vacant 3d orbitals.

Co^{3+} undergoing d^2sp^3 hybridization: 3d [↑↓ | ↑↓ | ↑↓ | |] [ ] [| |]

$[Co(NH_3)_6]^{3+}$ Inner orbital or low spin complex: 3d [↑↓ | ↑↓ | ↑↓]   [↑↓ | ↑↓ | ↑↓ | ↑↓ | ↑↓ | ↑↓]
d^2sp^3 hybrid orbitals
Six pairs of electrons from six NH_3 molecules

The six pairs of electrons, one from each NH_3 molecule, occupy the six hybrid orbitals. It proves that complex has octahedral geometry. Absence of unpaired electron makes this complex diamagnetic in nature.

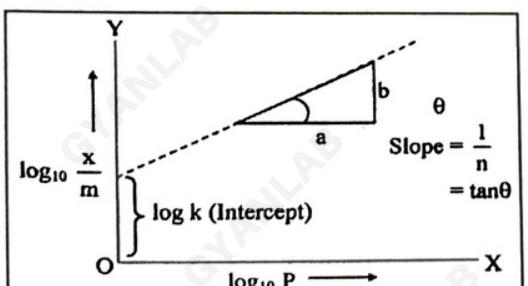


Structure of $[Co(NH_3)_6]^{3+}$

88. (C)
When hydrogen atoms of the amino group of arylamines are replaced by electron donating alkyl groups, the basic character of the resulting amine increases.
Benzenamine have highest pK_b value and is a weakest base in given compounds.

89. (C)
$Ni_{(s)} \mid Ni^{2\oplus}_{(1M)} \| Ag^{\oplus}_{(1M)} \mid Ag_{(s)}$
Oxidation Reduction
$\therefore$ Reducing agent = Ni

90. (B)



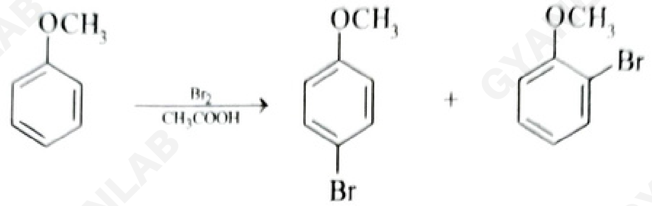
Variation of $\log_{10} x/m$ with $\log_{10} P$ according to Freundlich equation

91. (D)

92. (D)

$\overset{+4}{Sn}\overset{-1}{Cl_4} \Rightarrow Sn(IV)Cl_4$

93. (B)



Anisole   p-bromo anisole (90%)   o-bromo anisole (10%)

94. (D)

95. (C)

$$\begin{array}{c} CHO \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array} \xrightarrow[HNO_3]{(O)} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ COOH \end{array}$$

Glucose   Saccharic acid

96. (B)

$$\begin{array}{c} CHO \\ | \\ CHO \end{array}$$

Glyoxal   IUPAC name : Ethanedial

97. (C)

98. (A)

99. (B)

$$CH_3Br + AgNO_2 \longrightarrow \underset{(A)}{CH_3NO_2} \xrightarrow{Sn/HCl} \underset{(B)}{CH_3NH_2}$$

100. (B)

According to Boyle's law

$P_1V_1 = P_2V_2$

$1.013 \times 10^5 \times 2.27 = P_2 \times 4.54$

$\therefore\ P_2 = \dfrac{1.013 \times 10^5 \times 2.27}{4.54} = 5.065 \times 10^4\ N\ m^{-2}$

Section II

MATHEMATICS

101.(B)
We have $A \equiv (3, 2, -1)$, $B \equiv (-2, 2, -3)$ and $D \equiv (-2, 5, -4)$

$\therefore \overline{AB} = -5\hat{i} - 2\hat{k}$ and $\overline{AD} = -5\hat{i} + 3\hat{j} - 3\hat{k}$

Area of parallelogram $= |\overline{AB} \times \overline{AD}|$

Now $\overline{AB} \times \overline{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 0 & -2 \\ -5 & 3 & -3 \end{vmatrix} = \hat{i}(6) - \hat{j}(5) + \hat{k}(-15) = 6\hat{i} - 5\hat{j} - 15\hat{k}$

$\therefore$ Area $= \sqrt{(6)^2 + (-5)^2 + (-15)^2} = \sqrt{286}$ sq. units

102.(C)
We have $(\bar{x} - \hat{a}) \cdot (\bar{x} + \hat{a}) = 8$

$\therefore |\bar{x}|^2 - |\hat{a}|^2 = 8 \Rightarrow |\bar{x}|^2 = 8 + 1 = 9 \Rightarrow |\bar{x}| = 3$

103.(D)
We have $\ell(OP) = 4$ and $m\angle POX = 30°$

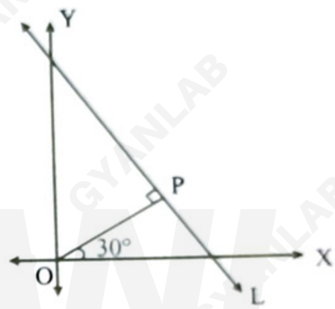
$\therefore P \equiv (4\cos 30°, 4\sin 30°) \equiv (2\sqrt{3}, 2)$

From figure, we conclude that angle made be line with +ve X axis is 120°

$\therefore$ Slope of line $= \tan(120°) = -\sqrt{3}$

Hence required equation of line L is

$(y-2) = (-\sqrt{3})(x - 2\sqrt{3}) \Rightarrow \sqrt{3}x + y = 8$



104.(A)

Let $I = \int \frac{\sin x}{\sin(x-\alpha)} dx$

$= \int \frac{\sin[(x-\alpha)+\alpha]}{\sin(x-\alpha)} dx = \int \frac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)} dx$

$= \cos\alpha \int dx + \sin\alpha \int \frac{\cos(x-\alpha)}{\sin(x-\alpha)} dx = (\cos\alpha)(x) + (\sin\alpha)\log|\sin(x-\alpha)| + c$

Comparing with given data, we get $A = \cos\alpha$ and $B = \sin\alpha$

105.(D)

Let p = Probability of getting a doublet in a throw of pair of dice.

$\therefore p = \frac{6}{36} = \frac{1}{6}$ and $q = 1 - \frac{1}{6} = \frac{5}{6}$

Dice are thrown 4 times.

Let X = 0, 1, 2, 3, 4 denote number of times a doublet is obtained.

When $X = 0$, $p = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{625}{1296}$

When $X = 1$, $p = \left({}^4C_1\right)\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{125}{324}$

When $X = 2$, $p = \left({}^4C_2\right)\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{25}{216}$

When $X = 3$, $p = \left({}^4C_3\right)\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6} = \frac{5}{324}$

When $X = 4$, $p = \frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6} = \frac{1}{1296}$

106.(D)

$$\bar{v}\times\bar{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 1 & 0 & 3 \end{vmatrix} = \hat{i}(6)-\hat{j}(7)+\hat{k}(-2) = 6\hat{i}-7\hat{j}-2\hat{k}$$

$\therefore\ |\bar{v}\times\bar{w}| = \sqrt{(6)^2+(7)^2+(2)^2} = \sqrt{89}$

Also $|\bar{u}| = 1$ ...(given)

$\therefore\ [\bar{u}\ \ \bar{v}\ \ \bar{w}] = \sqrt{89}$

107.(C)

Let p : -7 is an integer.

q : $\sqrt{-7}$ is a complex number.

Logical form of $S_1 \equiv p \rightarrow q$

Logical form of $S_2 \equiv \sim p \vee q$

We know that $p \rightarrow q \equiv \sim p \vee q$. $\therefore$ S_1 and S_2 are equivalent statements.

108.(C)

We have $0.07 + 0.2 + 0.3 + k + 0.07 + 0.04 + 0.02 = 1$

$\therefore\ k = 0.3$

x_i	p_i	p_ix_i	$p_ix_i^2$
5	0.07	0.35	1.75
6	0.2	1.2	7.2
7	0.3	2.1	14.7
8	0.3	2.4	19.2
9	0.07	0.63	5.67
10	0.04	0.4	4
11	0.02	0.22	2.42
Total		7.3	54.94

Variance (x) $= \Sigma p_ix_i^2 - \left(\Sigma p_ix_i\right)^2 = (54.94)-(7.3)^2 = 1.65$

109.(A)

Let $I = \int_0^1 |5x-3|\,dx$

$5x - 3 = 0 \Rightarrow x = \frac{3}{5}$

$$\therefore I = \int_0^{\frac{3}{5}} -(5x-3)\,dx + \int_{\frac{3}{5}}^{1} (5x-3)\,dx$$

$$= \frac{-5}{2}\left[x^2\right]_0^{\frac{3}{5}} + 3[x]_0^{\frac{3}{5}} + \frac{5}{2}\left[x^2\right]_{\frac{3}{5}}^{1} - 3[x]_{\frac{3}{5}}^{1}$$

$$= \left(\frac{-5}{2}\right)\left(\frac{9}{25}\right) + 3\left(\frac{3}{5}\right) + \frac{5}{2}\left(1-\frac{9}{25}\right) - 3\left(1-\frac{3}{5}\right)$$

$$= \frac{-45}{50} + \frac{9}{5} + \left(\frac{5}{2}\right)\left(\frac{16}{25}\right) - 3\left(\frac{2}{5}\right) = \frac{-45}{50} + \frac{3}{5} + \frac{8}{5} = \frac{-45+110}{50} = \frac{65}{50} = \frac{13}{10}$$

110.(D)

$$f(x) = e^{-\frac{1}{x}}$$

$$f'(x) = e^{-\frac{1}{x}}(-1)\left(\frac{-1}{x^2}\right) = \frac{1}{x^2 e^{\frac{1}{x}}}$$

When $f'(x) > 0,\ x^2 e^{\frac{1}{x}} > 0$ and $x \neq 0$.

Now $x^2 > 0$ and $e^{\frac{1}{x}} > 0$ for $\forall x \in R$.

Hence f(x) is an increasing function for $\forall$ x, except x = 0.

111.(B)

Let $\bar{a}_1 = 2\hat{i}+4\hat{j}+\hat{k}$ and $\bar{a}_2 = \hat{i}-2\hat{j}-3\hat{k}$

$\therefore\ \bar{a}_2 - \bar{a}_1 = -\hat{i}-6\hat{j}-4\hat{k}$ and let $\bar{b} = 3\hat{i}+5\hat{j}+2\hat{k}$

$$\bar{b}\times|\bar{a}_2-\bar{a}_1| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ -1 & -6 & -4 \end{vmatrix} = \hat{i}(-20+12) - \hat{j}(-12+2) + \hat{k}(-18+5) = -8\hat{i}+10\hat{j}-13\hat{k}$$

$$\therefore \left|\bar{b}\times(\bar{a}_2-\bar{a}_1)\right| = \sqrt{64+100+169} = \sqrt{333}$$

Also $|\bar{b}| = \sqrt{9+25+4} = \sqrt{38}$

$\therefore$ Distance between given lines $= \sqrt{\frac{333}{38}}$ units.

112.(C)

$$\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x-1} = 0$$

$$\therefore \frac{dy}{dx} = -\left(\frac{y^2+y+1}{x^2+x-1}\right)$$

$$\therefore \int \frac{dy}{y^2+y+1} = -\int \frac{dx}{x^2+x+1} \Rightarrow \int \frac{dy}{y^2+y+\frac{1}{4}+\frac{3}{4}} = -\int \frac{dx}{x^2+x+\frac{1}{4}+\frac{3}{4}}$$

$$\int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = -\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left[\frac{y+\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)}\right] = \frac{-1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left[\frac{x+\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)}\right]+C_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = \frac{-2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\frac{(2y+1)}{\sqrt{3}}+\left(\frac{2x+1}{\sqrt{3}}\right)}{1-\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}\right] = C_1 \Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\frac{2(x+y+1)}{\sqrt{3}}}{\frac{3-(4yx+2x+2y+1)}{3}}\right] = C_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{2(x+y+1)}{\sqrt{3}}\times\frac{3}{2(1-2xy-x-y)}\right] = C_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\sqrt{3}(x+y+1)}{1-2xy-x-y}\right] = C_1 \Rightarrow \tan^{-1}\left[\frac{\sqrt{3}(x+y+1)}{1-x-y-2xy}\right] = C_2 \quad \ldots\left[\text{where } C_2 = \frac{\sqrt{3}C_1}{2}\right]$$

$$\therefore \frac{\sqrt{3}(x+y+1)}{1-x-y-2xy} = \tan C_2 \Rightarrow \frac{x+y+1}{1-x-y-2xy} = \frac{\tan C_2}{\sqrt{3}} = c \quad \ldots\text{(say)}$$

$$\therefore x+y+1 = c(1-x-y-2xy)$$

113.(A)

$$\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right] = \tan\left[\tan^{-1}\left(\frac{\left(\frac{3}{4}\right)+\left(\frac{2}{3}\right)}{1-\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)}\right)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{17}{6}\right)\right] = \frac{17}{6}$$

114.(D)

$$\frac{dx}{dt} = \frac{x\log x}{t}$$

$$\therefore \int\frac{dx}{x\log x} = \int\frac{dt}{t}$$

$$\therefore \log|\log x| = \log|t| + \log c$$

$$\therefore \log x = tc \Rightarrow x = e^{tc}$$

115.(B)

$3 \sin \theta = 2 \sin 3\theta$

$= 2(3 \sin \theta - 4 \sin^3 \theta)$

$\therefore 8 \sin^3 \theta - 3 \sin \theta = 0$

$\therefore \sin \theta (8 \sin^2 \theta - 3) = 0$

$\therefore \sin \theta = 0$ or $\sin \theta = \pm \sqrt{\frac{3}{8}} = \pm \frac{\sqrt{3}}{2\sqrt{2}}$

Since, $0 < \theta < \pi$, we write $\sin \theta = \frac{\sqrt{3}}{2\sqrt{2}}$

116.(A)

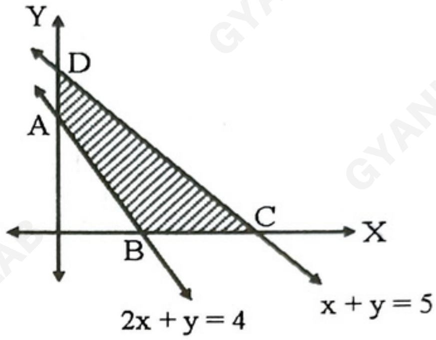
Refer figure

Required part is shaded.

We have A = (0, 4);

B = (2, 0); C = (5, 0);

D = (0, 5)



We have to maximize fuction

$z = 2x + 3y$

$\therefore z_A = 2(0) + 3(4) = 12$

$z_B = 2(2) + 3(0) = 4$

$z_C = 2(5) + 3(0) = 10$

$z_D = 2(0) + 3(5) = 15$

117.(A)

$(x + y)dy + (x - y)\, dx = 0$

We have $\frac{dy}{dx} = -\frac{(x-y)}{x+y}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{-(x-vx)}{x+vx} = \frac{-x(1-v)}{x(1+v)} = \frac{-(1-v)}{1+v}$

$\therefore x\frac{dv}{dx} = \frac{-1+v}{1+v} - v = \frac{-1+v-v-v^2}{1+v} \Rightarrow x\frac{dv}{dx} = \frac{-(1+v^2)}{1+v}$

$\therefore \int \frac{(1+v)}{1+v^2} dv = -\int \frac{dx}{x} \Rightarrow \int \frac{dv}{1+v^2} + \frac{1}{2}\int \frac{2v}{1+v^2} dv = -\int \frac{dx}{x}$

$\therefore \tan^{-1}(v) + \frac{1}{2}\log\left|1+v^2\right| = -\log x + \log c_1 \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right| = -\log x + \log c_1$

$2\tan^{-1}\left(\frac{y}{x}\right) + \log\left|\frac{x^2+y^2}{x^2}\right| + 2\log x = \log c$ ... $[\because \log c = 2\log c_1]$

$\therefore 2\tan^{-1}\left(\frac{y}{x}\right) + \log\left|\frac{x^2+y^2}{x^2} \times x^2\right| = \log c \Rightarrow 2\tan^{-1}\left(\frac{y}{x}\right) + \log\left|x^2+y^2\right| = \log c$

When $x = y = 1$, we get

$$2\tan^{-1}(1) + \log|2| \quad = \log c \quad \Rightarrow \log c = 2\left(\frac{\pi}{4}\right) + \log 2$$

$$\therefore \ \log|x^2 + y^2| \quad = \frac{\pi}{2} + \log 2 - 2\tan^{-1}\left(\frac{y}{x}\right) \Rightarrow \log\left|\frac{x^2+y^2}{2}\right| \quad = \frac{\pi}{2} - 2\tan^{-1}\left(\frac{y}{x}\right)$$

118.(C)

$$\text{Let} \quad I = \int \frac{10^{\frac{x}{2}}}{\sqrt{10^{-x} - 10^{x}}}\,dx$$

$$= \int \frac{10^{\frac{x}{2}}}{\sqrt{\frac{1}{10^x} - 10^x}}\,dx = \int \frac{10^{\frac{x}{2}}}{\sqrt{\frac{1-(10^x)^2}{10^x}}}\,dx = \int \frac{10^{\frac{x}{2}} \cdot 10^{\frac{x}{2}}}{\sqrt{1-(10^x)^2}}\,dx = \int \frac{10^x}{\sqrt{1-(10^x)^2}}\,dx$$

Put $10^x = t \quad \Rightarrow 10^x (\log 10)\,dx = dt$

$$\therefore \ I = \frac{1}{\log 10}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\log 10}\sin^{-1}(t) + c = \frac{1}{\log 10}\sin^{-1}\left(10^x\right) + c$$

119.(D)

$$\text{Let} \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{3\cos x + \sin x}\,dx$$

$$\text{Put} \quad \cos x = A(3\cos x + \sin x) + B\frac{d}{dx}(3\cos x + \sin x)$$

$$= A(3\cos x + \sin x) + B(-3\sin x + \cos x)$$

$$= \cos x\,(3A + B) + \sin x\,(A - 3B)$$

Thus $3A + B = 1$ and $A - 3B = 0$

Solving, we get $B = \frac{1}{10}$, $A = \frac{3}{10}$

$$\therefore \ I = \int_0^{\frac{\pi}{2}} \frac{\frac{3}{10}(3\cos x + \sin x) + \frac{1}{10}(-3\sin x + \cos x)}{3\cos x + \sin x}\,dx$$

$$= \frac{3}{10}\int_0^{\frac{\pi}{2}} dx + \frac{1}{10}\int_0^{\frac{\pi}{2}} \frac{\frac{d}{dx}(3\cos x + \sin x)}{3\cos x + \sin x}\,dx$$

$$= \frac{3}{10}[x]_0^{\frac{\pi}{2}} + \frac{1}{10}\Big[\log|3\cos x + \sin x|\Big]_0^{\frac{\pi}{2}} \quad = \frac{3}{10}\left(\frac{\pi}{2}\right) + \frac{1}{10}[\log|1| - \log|3|]$$

$$= \frac{3\pi}{20} - \frac{1}{10}\log 3$$

120.(D)

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$|A| = -4 - 15 = -19 \text{ and adj } A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} \Rightarrow k = \frac{1}{19}$$

121.(B)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$\therefore |A| = 1(4-4) - 2(-4-2) + 3(-2-1) = 12 - 9 = 3$$

We know that A (adj A) = |A| I

$$\therefore A(\text{adj } A) = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

122.(A)

We know that

$$k + 2k + 3k + 4k + 4k + 3k + 2k + k + k = 1 \Rightarrow 21k = 1 \Rightarrow k = \frac{1}{21}$$

When $x = 4, P = 4k = \frac{4}{21}$, When $x = 5, P = 3k = \frac{3}{21}$, When $x = 6, P = 2k = \frac{2}{21}$

$$\therefore P(3 < x \le 6) = \frac{4+3+2}{21} = \frac{9}{21} = \frac{3}{7}$$

123.(A)

$$y = x^3 - \alpha x^2 - \beta x + 5$$

$$\therefore \frac{dy}{dx} = 3x^2 - 2\alpha x - \beta$$

We have $x = -2$ and $x = 4$ as extreme points.

$$\therefore \left(\frac{dy}{dx}\right)_{x=-2} = 3(-2)^2 - 2\alpha(-2) - \beta = 0$$

$$\therefore 12 + 4\alpha - \beta = 0 \Rightarrow 4\alpha - \beta = -12 \quad \ldots(1)$$

$$\left(\frac{dy}{dx}\right)_{x=4} = 3(4)^2 - 2\alpha(4) - \beta = 0$$

$$\therefore 48 - 8\alpha - \beta = 0 \Rightarrow 8\alpha + \beta = 48 \quad \ldots(2)$$

Solving eq. (1) and (2), we get

$\alpha = 3, \beta = 24$

124.(A)

$$2x + y - 2z = 18$$

Dividing both sides by $\sqrt{(2)^2 + (1)^2 + (-2)^2} = 3$, we get

$$\left(\frac{2}{3}\right)x+\left(\frac{1}{3}\right)y-\left(\frac{2}{3}\right)z=6$$

Hence length of perpendicular from origin to the plane is 6.

Therefore coordinates of foot of perpendicular are $\left[\left(\frac{2}{3}\right)(6),\left(\frac{1}{3}\right)(6),\left(\frac{-2}{3}\right)(6)\right]$ i.e. $(4, 2, -4)$

125.(B)

Let (h, k) be the centre of circle.

$$\therefore \sqrt{(h-0)^2+(k-0)^2}=\sqrt{(h-x)^2+(k-0)^2}=\sqrt{(h-0)^2+(k-y)^2}$$

$$\therefore h^2+k^2=(h-x)^2+k^2=h^2+(k-y)^2$$

$$\therefore -2hx+x^2=0 \Rightarrow x(x-2h)=0 \quad \text{and}$$

$$-2ky+y^2=0 \Rightarrow y(y-2k)=0$$

$$\therefore x=0, 2h \quad \text{and} \quad y=0, 2k$$

$$\therefore x=2h \text{ and } y=2k, \Rightarrow h=\frac{x}{2}, k=\frac{y}{2}$$

126.(D)

$$\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 3 & 3 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$$

$R_3 \to R_3 - 3R_1$ and $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & -2\\ 0 & 6 & -7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}1\\ -1\\ -1\end{bmatrix}$$

$R_3 \to R_3 - 6R_2$

$$\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & -2\\ 0 & 0 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}1\\ -1\\ 5\end{bmatrix}$$

$$\therefore x_1-x_2+x_3=1$$

$$x_2-2x_3=-1$$

$$5x_3=5$$

$$\therefore x_3=1, x_2=1, x_1=1$$

127.(A)

$$(\bar{a}+\lambda\bar{b})\cdot\bar{c}=0$$

$$\therefore \left[(1-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right]\cdot\left[3\hat{i}+\hat{j}\right]=0$$

$$\therefore (1-\lambda)(3)+(2+2\lambda)(1)=0 \Rightarrow \lambda=5$$

128.(B)

We have $\tan\alpha+\tan\beta=\frac{-2h}{b}$ and $\tan\alpha\cdot\tan\beta=\frac{a}{b}$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{\left(\frac{-2h}{b}\right)}{1-\left(\frac{a}{b}\right)} = \frac{-2h}{b}\times\frac{b}{b-a} = \frac{2h}{a-b}$$

129.(D)

We have $\tan\alpha = \left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

Put $x^2 = \cos\theta$

$$\therefore \tan\alpha = \frac{\sqrt{2\cos^2\frac{\theta}{2}}-\sqrt{2\sin^2\frac{\theta}{2}}}{\sqrt{2\cos^2\frac{\theta}{2}}+\sqrt{2\sin^2\frac{\theta}{2}}}$$

$$\therefore \tan\alpha = \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}} = \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}} = \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$\therefore \alpha = \frac{\pi}{4}-\frac{\theta}{2} \Rightarrow 2\alpha = \frac{\pi}{2}-\theta$$

$$\therefore \sin 2\alpha = \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta = x^2$$

130.(B)

$$\frac{dy}{dx} = 2^{y-x}$$

$$\therefore \frac{dy}{dx} = \frac{2^y}{2^x} \Rightarrow \int\frac{dy}{2^y} = \int\frac{dx}{2^x}$$

$$\therefore \int 2^{-y}dy = \int 2^{-x}dx \Rightarrow \frac{2^{-y}}{-\ell n2} = \frac{2^{-x}}{-\ell n2}+c_1 \Rightarrow \frac{2^{-y}}{\ell n2} = \frac{2^{-x}}{\ell n2} - c_1$$

$$\therefore \frac{1}{\ell n2}\left[2^{-x}-2^{-y}\right] = c_1$$

$$\therefore \frac{1}{2^x}-\frac{1}{2^y} = c, \text{ where } c = (c_1)(\ell n 2)$$

131.(A)

d.r. of line through PQ are [(2 – 1), (3 – 2), (4 – 3)] i.e. (1, 1, 1)

Hence required equation of line is

$$\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$$

132.(A)

$f(x) = 3 + 2^x + 4^x = y$ …(say)

Let $2^x = a \Rightarrow 4^x = a^2$

$\therefore a^2 + a + (3 - y) = 0$

As a ∈ R, we write

$(1)^2 - 4(1)(3 - y) \geq 0$

$\therefore\ 1 - 12 + 4y \geq 0 \quad \Rightarrow 4y \geq 11 \quad \Rightarrow y \geq \frac{11}{4}$

Also $2^x + 4^x > 0 \quad \Rightarrow y \neq 3$

$\therefore$ Range from given options is $(3, \infty)$

133.(B)

The equations of two required lines are $y = \sqrt{3}\,x$ and $y = \frac{1}{\sqrt{3}}x$

Their combined equation is

$(\sqrt{3}x - y)(x - \sqrt{3}\,y) = 0$ i.e. $\sqrt{3}\,x^2 - 4xy + \sqrt{3}y^2 = 0$

134.(C)

Let $I = \int e^{e^x + x}\,dx = \int e^{e^x} \cdot e^x\,dx$

Put $e^x = t \Rightarrow e^x\,dx = dt$

$\therefore\ I = \int e^t\,dt = e^t + c = e^{e^x} + c$

135.(B)

Let $z = x + iy$ and $|z + 1| = 1$

$\therefore\ |(x + 1) + iy| = 1$

$\therefore\ \sqrt{(x+1)^2 + y^2} = 1 \Rightarrow (x + 1)^2 + y^2 = 1$

This is an equation of a circle with centre $(-1, 0)$ and radius 1.

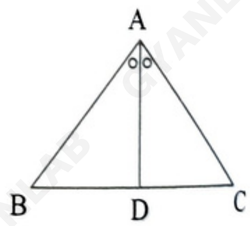
136.(D)

We have $\bar{a} = 3\hat{j}$, $\bar{b} = 4\hat{k}$ and $\bar{c} = 3\hat{j} + 4\hat{k}$

Angle bisector AD divides BC in the ratio AB : AC.

$AB = \sqrt{9+16} = 5$ and $AC = \sqrt{0+16} = 4$

Thus D divides BC in the ratio 5 : 4



$\therefore\ D = \left[0, \frac{5(3)}{5+4}, \frac{5(4)+4(4)}{5+4}\right]$

$= \left(0, \frac{15}{9}, \frac{20+16}{9}\right) = \left(0, \frac{5}{3}, 4\right) \Rightarrow \frac{5}{3}\hat{j} + 4\hat{k}$

137.(A)

Let the two parts of 10 be x and $(10 - x)$.

$f(x) = 2(10 - x) + x^2 = x^2 - 2x + 20$

$f'(x) = 2x - 2$ and when $f'(x) = 0$, we get $x = 1$

$f''(x) = 2 > 0$

$\therefore$ f(x) is minimum at x = 1.

Thus the parts are 1, 9.

138.(A)

$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

$$= \frac{b^2+c^2-a^2}{(2bc)(a)} + \frac{c^2+a^2-b^2}{(2ac)(b)} + \frac{a^2+b^2-c^2}{(2ab)(c)} = \frac{b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2}{2abc}$$

$$= \frac{a^a+b^2+c^2}{2abc} = \frac{4+9+25}{2(2)(3)(5)} = \frac{38}{60} = \frac{19}{30}$$

139.(B)

$f(x) = \frac{(1+\cos x)-(\sin x)}{(1+\cos x)+(\sin x)}$, is continuous at $x = \pi$.

$$\lim_{x\to\pi} f(x) = \lim_{x\to\pi} \frac{\left(2\cos^2\frac{x}{2}\right)-\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\left(2\cos^2\frac{x}{2}\right)+\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)} = \lim_{x\to\pi} \frac{2\cos\frac{x}{2}\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{2\cos\frac{x}{2}\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}$$

$$= \lim_{x\to\pi} \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}} = \lim_{x\to\pi} \tan\left(\frac{\pi}{4}-\frac{x}{2}\right) = \tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right) = -1$$

140.(D)

We have $n = 16$, $\Sigma x_i = 528$, and $\frac{528}{16} = 33 \Rightarrow \bar{x} = 33$ and $\Sigma(x_i - \bar{x})^2 = 9158$

Variance $= \frac{1}{n}\Sigma(x_i-\bar{x})^2 = \frac{9158}{16} = 572.375$

141.(A)

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\log x}$$

$$= \lim_{x\to 1} \frac{\frac{1}{2\sqrt{x}}}{\left(\frac{1}{x}\right)} \qquad \text{… [L' Hospital Rule]}$$

$$= \frac{1}{2}$$

142.(D)

By Newton's law of cooling, we write

$$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$\therefore \frac{d\theta}{dt} = k(\theta - \theta_0) \Rightarrow \int\left(\frac{d\theta}{\theta-\theta_0}\right) = \int k\,dt$

$\therefore \log|\theta - \theta_0| = kt + c$ …(1)

When $t = 0$, $\theta = 80$ and $\theta_0 = 25$

$\therefore \log|80 - 25| = 0 + c \Rightarrow c = \log|55|$ …(2)

When $t = 30$, $\theta = 50$

$\therefore \log|50 - 25| = 30k + \log|55|$

$\therefore\ k = \frac{1}{30}\log\left|\frac{5}{11}\right|$ ...(3)

From (1), (2), (3) we write

$\log|\theta - \theta_0| = \frac{1}{30}\log\left|\frac{5}{11}\right| t + \log|55|$

When t = 60, we get

$\log|\theta - \theta_0| = \frac{60}{30}\ \log\left|\frac{5}{11}\right| + \log|55| = \log\left(\left|\frac{5}{11}\right|\right)^2 + \log|55| = \log\left|\frac{25}{121}\times 55\right| = \log\left|\frac{125}{11}\right|$

$\therefore\ \theta - 25 = \frac{125}{11} \Rightarrow \theta = 36.36°\ C$

143.(D)

Prime numbers from 1 to 19 are 2, 3, 5, 7, 11, 13, 17, 19 i.e. 8 such numbers.

The room with prime number is given to the first guest.

$\therefore$ The probability that second guest will get a room with prime number = $\frac{7}{18}$.

144.(C)

Let p : $3 + 6 > 8$ and q : $2 + 3 < 6$

The logical form of given statement is $p \wedge q$.

$\therefore\ \sim(p \wedge q) \equiv \sim p \vee \sim q$ i.e. $3 + 6 \le 8$ or $2 + 3 \ge 6$

145.(A)

$$f(x) = \operatorname{cosec}^{-1}\left[\frac{10}{6\sin(2^x) - 8\cos(2^x)}\right] = \sin^{-1}\left[\frac{6\sin(2^x) - 8\cos(2^x)}{10}\right]$$

Here $(6)^2 + (-8)^2 = (10)^2$

$\therefore$ Let $\cos\alpha = \frac{6}{10}$ and $\sin\alpha = \frac{8}{10}$

$\therefore\ f(x) = \sin^{-1}\left[\sin(2^x)\cos\alpha - \cos(2^x)\sin\alpha\right] = \sin^{-1}[\sin(2^x - \alpha)]$

$\therefore\ f(x) = 2^x - \alpha \Rightarrow f'(x) = 2^x \log 2$

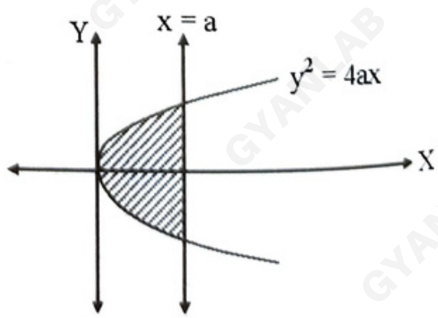
146.(A)

Refer figure.

Required area is shaded.

Point of intersection of $x = a$ and $y^2 = 4ax$, is

$y^2 = 4a^2 \Rightarrow y = \pm 2a$ and $x = a \Rightarrow (a, \pm 2a)$



$\therefore\ A = 2\int_0^a \left(2\sqrt{a}\sqrt{x}\right)dx$

$$= 4\sqrt{a}\int_0^a x^{\frac{1}{2}}\,dx = 4\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^a = \left(4\sqrt{a}\right)\left(\frac{2}{3}\right)\left(a\sqrt{a}\right) = \frac{8}{3}a^2 \text{ sq. units}$$

147.(A)

$y = \log\sqrt{\tan x}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\sqrt{\tan x}} \times \dfrac{1}{2\sqrt{\tan x}} \times \sec^2 x = \dfrac{\sec^2 x}{2\tan x}$

$\therefore \left(\dfrac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \dfrac{\left(\sqrt{2}\right)^2}{2} = 1$

148.(D)

$x = a\left(t - \dfrac{1}{t}\right)$ and $y = b\left(t + \dfrac{1}{t}\right)$ ...(1)

$\therefore \dfrac{dx}{dt} = a\left(1 + \dfrac{1}{t^2}\right)$ and $\dfrac{dy}{dt} = b\left(1 - \dfrac{1}{t^2}\right)$

$\therefore \dfrac{dy}{dx} = \dfrac{b\left(1 - \frac{1}{t^2}\right)}{a\left(1 + \frac{1}{t^2}\right)} = \left(\dfrac{b}{a}\right)\left(\dfrac{t^2 - 1}{t^2 + 1}\right)$ ...(2)

Now $x = a\left(\dfrac{t^2 - 1}{t}\right)$ and $y = b\left(\dfrac{t^2 + 1}{t}\right)$ ...[From (1)]

$\therefore (t^2 - 1) = \left(\dfrac{x}{a}\right)t$ and $(t^2 + 1) = \left(\dfrac{y}{b}\right)(t)$

$\therefore$ Eq. (2) becomes

$\dfrac{dy}{dx} = \left(\dfrac{b}{a}\right)\left(\dfrac{x}{a}\right)(t) \times \dfrac{1}{\left(\frac{y}{b}\right)\times t} = \dfrac{b^2 x}{a^2 y}$

149.(A)

We have to choose 3 consonants and 2 vowels from 7 consonants and 4 vowels.

$\therefore$ Number of words $= \left({}^7C_3\right) \times \left({}^4C_2\right) \times 5!$

$= \dfrac{7!}{3!4!} \times \dfrac{4!}{2!2!} \times 5! = 25200$

150.(B)

The equation of planes parallel to the plane $x - 2y + 2z + 4 = 0$ is

$x - 2y + 2z + \lambda = 0$ .... (i)

The required planes are at a distance of one unit from the point (1, 2, 3)

$\left|\dfrac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| = 1$

$\left|\dfrac{1(1) + (-2)(2) + 2(3) + \lambda}{\sqrt{1+4+4}}\right| = 1 \Rightarrow \left|\dfrac{1 - 4 + 6 + \lambda}{3}\right| = 1 \Rightarrow 3 + \lambda = \pm 3$

$\therefore$ $3 + \lambda = 3$ or $3 + \lambda = -3$

$\lambda = 0$ or $\lambda = -6$

The equation of planes are $x - 2y + 2z = 0$ and $x - 2y + 2z - 6 = 0$